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14. ABSTRACT

We have continued to work on control of complex nonlinear systems and pursued applications to control of mixing, control of nanoscale processes and control of microscale mixing. At the PI's group, 1 graduate student (Umesh Vaidya) was working on the problem of control of discrete-time, conservative systems and quantum control. Another student (Zoran Levnajic) worked on problems in visualization of dynamical systems. A postdoctoral student (Dmitri Vainchtein) continued working on the problem of flow control using tools from dynamical systems theory and vortex dynamics and pursued a problem in control of nanoparticial seperation. The PI worked on ergodic theory methods for control of systems with drift and optimazation of mixing in a paper appeared in the journal Nature. The PI also worked extending the framework for model validation of Random Dynamical Systems in the framework of the Koopman operator developed into dynamical systems treatment of uncertainty analysis.

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Final Technical Report for the Grant: Nonlinear Dynamics and Ergodic Theory Methods in Control AFOSR grant F49620-00-1-0361

Igor Mezić
Department of Mechanical and Environmental Engineering,
University of California, Santa Barbara

2000-2003

1   Objectives

The objective of our research was to develop new control theory and dynamical systems/ergodic theory tools for the problems of active control of complex fluid systems and control of mixing in fluids and apply the developed theoretical tools to problems of active control of mixing in combustion chambers of jet engines.

2   Timeline of Effort

2.1   2000-2001

We have continued the work (started in a previous AFOSR-sponsored research) on control of complex nonlinear systems and pursued applications to fluid flow control. At the PI's group, 1 graduate student (Umesh Vaidya) was working on the problem of linking issues in KAM theory and small input control theory. A postdoc (Dmitri Vainchtein) worked on the problem of vortex merger control using tools from averaging theory. The PI worked on aspects of controllability of group translations that had ramifications for both of the previously mentioned problems and collaborated with Bassam Bamieh (UCSB) on mixing by optimal destabilization. The PI pursued a collaboration with a number of experimental groups (F. Sotiropoulos at Georgia Tech and Tom Solomon at Bucknell University) on theoretical, computational and experimental aspects of mixing. There was an active exchange of information with researchers from the United Technologies Research Center.

The new developments in this period were: 1) Development of a new approach to controllability of Hamiltonian systems using ergodic theory/group theory tools, [13, 28] 2) Study of control of vortex merger using a combination of dynamical systems/control theory techniques [32], 3) A framework for optimal destabilization of dynamical systems [2] 4) Development of theoretical framework for experimental studies of mixing [18].

Overall this year we developed a promising approach to control of flows that spun off ideas for development of techniques in control of Hamiltonian systems. These ideas led to work on fundamental aspects of group translation controllability which in turn told us how to harness the internal dynamics of a fluid flow in order to achieve an objective. At all times we were led by the key aspect of the physical problem we were interested in: control of vortex merger.

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2.2   2001-2002

We have continued the work on control of complex nonlinear systems and pursued applications to fluid flow control. At the PI's group, 1 graduate student (Umesh Vaidya) was working on the problem of in Kolmogorov-Arnold-Moser theory of three-dimensional, volume-preserving flows [29]. A postdoc (Dmitri Vainchtein) worked on the problem of flow control using tools from dynamical systems theory [33]. The problem of dynamical systems with moving averages was considered in [36]. The PI worked on problems of model validation of Random Dynamical Systems in the framework of the Koopman operator [16] and also applied ideas from control of mixing to a design of a pasive micromixer in collaboration with an experimental group [26], in a paper that apearred in the journal Science. There was an active exchange of information with researchers from the United Technologies Research Center on model validation, mixing in combustion chambers and jet noise.

There was some unification of different topics into a framework: the study of KAM theory in three dimensions in [29] is is linked to control-theoretic results of the type we reported in two dimensional maps in [28]. This in turn is important for mixing in three-dimensional flows of the type occuring in combustion chambers. The extension of previous results on model validation to Random Dynamical Systems [16] opens the door to a diverse set of applications of these techniques. These results are arrived at in a unified framework of Random Dynamical Systems which promises to be useful in other areas such as uncertainty analysis.

2.3   2002-2003

We have continued the work on control of complex nonlinear systems and pursued applications to control of mixing, control of nanoscale processes and control of microscale mixing. At the PI's group, 1 graduate student (Umesh Vaidya) was working on the problem of control of discrete-time, conservative systems [30, 31] and quantum control [27]. Another student (Zoran Levnajić) worked on problems in visualization of dynamical systems [9]. A postdoc (Dmitri Vainchtein) continued working on the problem of flow control using tools from dynamical systems theory and vortex dynamics [35, 33] and pursued a problem in control of nanoparticle separation [34]. The PI worked on ergodic theory methods for control of systems with drift [14, 15] and optimization of mixing in a paper appeared in the journal *Nature* [25]. The PI also worked on extending the framework for model validation of Random Dynamical Systems in the framework of the Koopman operator developed in [16] to dynamical systems treatment of uncertainty analysis [17]. There was an active exchange of information with researchers from the United Technologies Research Center on uncertainty analysis, bluff body combusting flow control [11] micromixing and jet noise.

3   Accomplishments

The research achievements of the project have been:

3.1   Group theory, the Anti-KAM results and controllability of Hamiltonian systems

In [13] we pursued the question of controllability of group translations. Systems theory on Lie groups [3][8] is a well-developed and beautiful subject relying on differential-geometric and algebraic methods. In [13] we utilize some ergodic theory concepts that seem to be useful for the analysis of controllability for translations on abstract groups. These results in turn have consequences for controllability of linear systems with input saturation, control of chaos and control of mixing in fluids.

As a motivation, we ask the following simple question: Let T be the translation of the circle S^1 such that $T : \theta \rightarrow \theta + \omega$ and at each discrete time step ω can be chosen from a set $U \subset [0, 1)$.

Assuming the iteration starts from θ_0, can every θ_1 on the circle be reached in finite time by the appropriate choice of ω's, i.e. is it true that for every $\theta_1, \theta_0 \in S^1$ there exist n, $\omega_i \in U$ such that $\theta_1 = \theta_0 + \sum_{i=1}^{n} \omega_i$. If yes, T is called controllable. Notice that without loss of generality we can set $\theta_0 = 0$ and thus instead consider the question of reachability from 0. We analyzed controllability and found an interesting link between ergodic properties of translations on groups and their controllability. In particular, if a compact group has an ergodic element (i.e. an element such that group translation under that element is an ergodic dynamical system) than with any input set U of positive measure that contains the ergodic element the group translation is controllable. These results can be extended to non-compact groups that are covering spaces for compact groups. We treat the case of $\mathbb{R}^n$ using the results on torus translations to obtain results on the controllability of $x \to x + y$, where $x \in \mathbb{R}^n, y \in U \subset \mathbb{R}^n$.

The general theme that needs to be stressed is that if the system has sufficiently complicated (say ergodic - which implies almost controllable) dynamics with some inputs, it is enough to include one of these inputs in the set of possible inputs which in addition needs to be of positive measure in order to get controllability. The relationship between ergodicity and controllability is interesting and points to further exploration of connections between ergodic theory and control theory previously pursued in [6, 1, 5].

There are two connections with applied issues that are of interest: there has been a significant amount of work in the physics community on the so-called Ott-Grebogi-Yorke (OGY) control algorhitm [22, 20] where the ergodic properties of the attractor are used to bring the system close to the desired location. The paper [13] provides a rigorous template of ideas upon which a mathematical theory of OGY-type approaches can be built. These ideas in turn seem to be of crucial importance in control of fluid flows, where the central issue is achieving an improvement in performance with small control inputs. In order for this to be possible, natural instabilities of the system need to be exploited. This is related to the global ergodic properties of the system and fits within the kind of analysis that we pursued.

Techniques developed in [13] can be used in much more complicated control situations than group translation control. For example, in [28, 30] we pursued a study of controllability under small input control of the following boundary and orientation-preserving discrete system on an annulus $[a, b] \times S^1$:

$$x_{t+1} = x_t + y_t + \epsilon u(t) f(x_t, y_t) \pmod{1}, \tag{1}$$

$$y_{t+1} = y_t + \epsilon u(t) g(x_t, y_t). \tag{2}$$

when $u(t) = 1$, this is a well-studied system. In particular, if $\epsilon = 0$, the dynamics is confined to invariant curves $y = const$. The so-called KAM (Kolmogorov-Arnold-Moser) theorem states that most of the invariant curves persist if the perturbation is sufficiently small. Using the techniques similar to those developed in [13] we prove that under mild conditions on f and g and $u \in [-1, 1]$ the system 1 is controllable. This result is constructive in the sense that u is given as feedback control. Such results can be extended to more general Hamiltonian system contexts. A particular application that we are currently pursuing is satellite formation control.

3.2 Control of vortex merger

The above ideas indicate a way of using system's internal dynamics to achieve a control objective with bounded input. A specific problem of great interest in fluid dynamics is that of vortex merger, that we studied in [35]. Two-dimensional incompressible flows tend to generate coherent vortical structures. In fact, many of the flows of AFOSR interest exhibit coherent vortices: diffuser flows, trailing airplane wing vortices, flow over a cavity etc. Vortices of same sign tend to rotate around each other and merge, producing tonal sound. In such situations flow control problem might necessitate enhancement or prevention of a vortex merger. We have studied some low-order modelling schemes for vortex merger control.

3

A vortex pair with the vortices of the same sign can be described by four variables: the position of the vorticity centroid, VC, $\mathbf{x}_c$, the vortex separation, VS, and the relative phase of vortex rotation, φ. The position of the VC is

$$\mathbf{x}_c = \frac{1}{\Gamma}\left(\Gamma_1 \mathbf{x}_1 + \Gamma_2 \mathbf{x}_2\right). \tag{3}$$

where Γ_i is the strength of the i-th vortex and $\Gamma = \Gamma_1 + \Gamma_2$. We restrict our discussion to the case of same sign vortices. The VS, r, is equal to the sum of the distances of the vortices 1 and 2 from the VC, r_i, $i = 1, 2$, respectively: $r = r_1 + r_2$.

In the absence of a control field (the unperturbed system) each vortex moves along the circle of radius r_i with the same angular frequency Ω, staying on opposite sides from the VC. The position of the VC and the VS are the integrals of the unperturbed system.

Suppose that an imposed perturbation (control) is small: a characteristical time scale of the unperturbed system is much smaller than a characteristical time scale of the perturbation. This approximation allows us to average the perturbation over a fast period and consider an averaged system instead of the exact one. In our particular case this approximation means that we assume that the drift of, say, a VC over a the period of vortex rotation is much smaller than the VS. Such an approach can be considered as a particular case of the singular perturbation methods [23]

Let $\varepsilon \ll 1$ be a characteristic amplitude of the perturbation. Then the evolution equations of the perturbed system has the following form:

$$
\begin{aligned}
\dot{x}_c &= \varepsilon f_1\left(\mathbf{x}_c, r, \varphi, t\right), \\
\dot{y}_c &= \varepsilon f_2\left(\mathbf{x}_c, r, \varphi, t\right), \\
\dot{r} &= \varepsilon f_3\left(\mathbf{x}_c, r, \varphi, t\right), \\
\dot{\varphi} &= \Omega\left(r\right) + \varepsilon f_4\left(\mathbf{x}_c, r, \varphi, t\right).
\end{aligned} \tag{4}
$$

In (4) the functions f_i are 2π-periodic in φ. It follows from (4) that the rate of change of φ is much larger (by a factor of order $1/\varepsilon$) than the rate of change of other variables. That is why we call φ a fast variable and $\mathbf{x}_c$ and r — slow variables.

Various actuation methods such as an external strain field and blowing/suction were studied leading to different functions f_i in 4. Averaging method was applied leading to results on optimal change of vortex separation. It was found, using Pontryagin maximum principle that a particular bang-bang scenario leads to optimal protocol in the blowing/suction actuation case.

3.3 Optimal destabilization of dynamical systems

There are a number of applied situations where the problem of interest involves destabilization of a solution of a dynamical system. Consider for example the problem of inverted pendulum. The stable equilibrium solution needs to be destabilized in order to get to the inverted position of the pendulum. A framework for solving such problems in an optimal way was considered in [2]. We consider the following LTI system which is subject to both exogenous disturbances w and controls u

$$
\begin{aligned}
\dot{x} &= Ax + B_1 w + B_2 u \\
z &= Cx
\end{aligned} \tag{5}
$$

The signal z represents the output of the system with the C matrix chosen such that the 2-norm of z represents the proper notion of "size" of output (or state).

The above is the formulation of the so-called standard problem in robust control, where the objective is to design the control u such that the "gain" of the system from w to z is minimized. The gain is usually measured in terms of the $\mathcal{H}^2$, $\mathcal{H}^\infty$ or ℓ^1 system norms. This problem can also be

viewed as a differential game in the sense that the disturbance w is trying to maximize $\|z\|$, while the control u is trying to minimize it.

The problem we are interested in is significantly different. In our formulation, it is desired that the gain from w to z be maximized rather than minimized. This of course is a trivial problem if there are no constraints on the size of the control. Indeed, with mild controllability assumptions on (A, B_2) it is always possible to design a feedback such that $\max_{w \neq 0} \frac{\|z\|}{\|w\|}$ is infinite. Therefore, a more meaningful statement of the problem is as follows

Design a feedback control for the system (5) such that the gain $\max_{w \neq 0} \frac{\|z\|}{\|w\|}$ is maximized, while keeping the control effort $\|u\|$ relatively small.

From the above, it is clear that the essence of this destabilization problem is the trade-off between enlarging the closed loop gain and keeping the control effort small. In regulation problems, the trade-off between regulation (minimizing the gain) and control effort is typically formulated by augmenting the control signal u into the regulated variables z. In our current problem, this is not applicable since the objective is to enlarge z while keeping u small.

It is possible however to have a linear quadratic formulation that captures the above problem statement as follows

Given a trade-off parameter α, design a feedback control for the system (5) such that u maximizes the quantity

$$\max_{w \neq 0} \frac{\|z\|^2}{\|w\|^2 + \alpha^2 \|u\|^2}, \tag{6}$$

subject to the dynamical constraints (5).

This formulation essentially amounts to adding the requirement that $\|u\|$ be small as a soft constraint. The above ratio is a lower bound on the ratios $\frac{\|z\|}{\|w\|}$ and $\frac{\|z\|}{\alpha\|u\|}$. Thus, the maximization of the ratio in (6) will insure that $\alpha\|u\|$ is small compared to $\|z\|$, and that the closed loop gain is large. The parameter α is then used to capture the trade-off, a large α would insure that the control effort is kept small, while a small α would maximize the gain at the cost of a potentially large $\|u\|$. It can be reformulated as

Given $\gamma > 0$, find u such that

$$\sup_{w \neq 0} \frac{\|z\|^2}{\|w\|^2 + \alpha^2 \|u\|^2} > \gamma^2$$

subject to the dynamical constraints (5).

We can convert this problem formulation to that of finding the worst case inputs w and u that maximize a quadratic objective. Over finite time horizons, this is clearly a well posed LQ problem, but over infinite time horizons, we expect feedback gains thus obtained to yield unbounded signals u and z. This is however not a conceptual difficulty since we are after the most destabilizing *feedback gains*. It turns out that even though the signals are unbounded as the time horizon goes to infinity, the feedback gains limit to a well defined gain. The solution to this problem was obtained in terms of a differential Ricatti equation.

3.4 Experimental visualization of mixing in vortex breakdown flows

In flows of practical importance it is important both from the perspective of understanding dynamics and designing control to have robust visualization tools for mixing. One such class of flows is those exhibiting vortex breakdown. In the work [18] we analyzed an experimental technique for constructing Poincaré maps in flows exhibiting chaotic advection and developed the theoretical framework based on ergodic theory that explains the reasons for the success of this approach. The technique is non-intrusive and, thus, simple to implement. Planar laser-induced fluorescence (LIF) is employed to collect a sufficiently long sequence of instantaneous light intensity fields on the plane of

section of the Poincare map (defined by the laser sheet). The invariant sets of the flow are visualized by time-averaging the instantaneous images and plotting iso-contours of the so resulting mean light intensity field. By linking the Eulerian time-averages of light intensity at fixed points in space with the Lagrangian time-averages along particle paths passing through these points, we showed that ergodic theory concepts can be used to show that this procedure will indeed visualize invariant sets of the Poincare map. As the technique is based on time-averaging, the rates of convergence are important: we showed that inside regular islands the convergence is fast.

3.5 KAM theory for three-dimensional, action-action-angle maps

An important class of three-dimensional volume-preserving maps and flows arises as a perturbation from integrable action-action-angle maps and flows. For example, three-dimensional, time dependent perturbations of vortex rings with no swirl lead to such Poincaré maps. We studied properties of this class of maps and flows. While action-angle-angle volume-preserving maps admit an analogue of the KAM theorem, general results on non-existence of two-dimensional invariant manifolds of action-action-angle maps are proven in [12]. Non-existence of such two-dimensional invariant manifolds means possibility of global transport and a mechanism for such transport - the local mechanism of resonance induced dispersion [4] - was studied perturbatively. Resonance induced dispersion was shown to arise from the existence of periodic orbits of saddle-focus type that survive perturbation at places where two-dimensional invariant manifolds break down. Still, in some flows of practical importance in combustion chambers, resonance-induced dispersion does not happen and stability of particle motion is seen. Our purpose is to understand the context in which this is possible.

In [29] we studied maps of the type

$$
\begin{aligned}
J_1' &= J_1 + \epsilon f_1(J_1, J_2, \phi), \\
J_2' &= J_2 + \epsilon f_2(J_1, J_2, \phi), \\
\phi' &= \phi + J_1 + \epsilon f_3(J_1, J_2, \phi).
\end{aligned}
\tag{7}
$$

This is an example of a three-dimensional, volume preserving map arising from periodically driven three-dimensional fluid flows such as perturbed vortex ring flows occuring in combustion chambers under pulsed flow control. In this example, in the integrable case (no pulsing and no swirl) J_2, ϕ should be thought of as azimuthal and cross-sectional angle, while J_1 is the torus-labeling coordinate. Our study indicates that such maps admits a KAM-type theorem on stability of particle motion. In particular, for small ϵ there is a large set of invariant tori that survive the perturbation and thus the hot core inside the vortex ring stays stable.

3.6 Control of vortex merger - the vortex patch case

A problem of great interest in flow control, due to its importance in shear flow dynamics (and thus for combusting flows, jet noise, etc.), is that of vortex merger. We studied the case when vortices are at a large distance compared with the core radius in [35]. The case when the vortices are in close proximity can be studied using the vortex patch model, as done in [33]. The configuration of two elliptical vortex patches is shown in figure 1.

In [33] the equations of motion for the aspect ratio and the orientation angle with control are developed to model the motion of identical patches, including forcing or preventing the merging, using a point vortex located at the center of the vorticity. We implemented two different approaches to this problem: using the method of flat coordinates (used for vortex control previously in [21]) and using averaging theory (extending the methods in [32]).
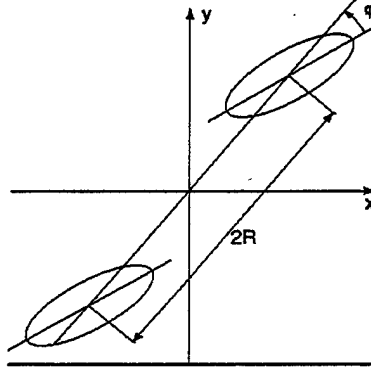
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Figure 1: A system of two elliptical vortex patches.

3.7 Model validation for Random Dynamical Systems

We developed a formalism for a class of stochastic systems - Random Dynamical Systems - in the context of Koopman operator akin to that of deterministic systems that allows for a systematic comparison of different models or data with stochastic elements. In this extension of the deterministic theory we studied deterministic factors of stochastic systems - a concept that might help in understanding e.g. the abundance of oscillatory phenomena on various time-scales in climate dynamics (see e.g. [24]). Our methods allow for model parameter identification in this context. They also allow for an easy distinction between processes having a deterministic factor on a circle (deterministic limit cycling) with additive noise, and lightly damped but stable (i.e. determistic factor has a fixed point) process - a question that received some interest in the combustion literature [10, 7].

On the applied side, the work on invariant measures described above can be used to distinguish between limit cycling systems and linear lightly damped systems with noise. For example, methods based on this theory were applied to the combustion instability problem at the United Technologies Research Center. A simple model of a combustion process is an inteconnection of a linear acoustic model and nonlinear heat release model that consists of a delay and a saturation function. The system is driven by broad-band stochastic disturbance. More precisely, a discrete-time model equations used to simulate pressure oscillations in the UTRC combustion rig were

$$x_{i+1}^1 = (-\alpha + \cos(\omega_0 T_s))x_i^1 - \sin(\omega_0 T_s)x_i^2,$$
$$x_{i+1}^2 = \sin(\omega_0 T_s)x_i^1 + (-\alpha + \cos(\omega_0 T_s))x_i^2 + K_3 h(K_2 x_{i-N}^1) + K_1 n_i, \qquad (8)$$

where $T_s = 0.0005$, $\omega_0 = 2\pi f_0$, $K_3 = 0.0525$, and h is a saturation function defined as $h(u) = u$ for $-s < u < s$, $h(u) = -s$ for $u \leq -s$, and $h(u) = s$ for $s \leq u$. Variables x_i^1, x_i^2 are unsteady components of pressure in the combustor at two different times, while variable n_i represents noise. The model was implemented in Simulink. To simulate noise, a Simulink model of a band-limited white noise with power 0.01 was used. The model described by (8) is a Discrete Random Dynamical System (DRDS), in a class for which we developed theoretical results described above. We choose a 2-dimensional embedding space for the system (which is formally $N + 2$-dimensional).

To obtain the harmonic averages 20,000 samples (10 seconds sampled at 2 kHz) of experimentally obtained combustor pressure and pressure from Simulink model simulations were used. The experimental data presents a spectrum with a single peak at about $f = 207$. We examined the results of harmonic analysis results for a range of model parameters lead by this spectral information. Values of f_0, N, α, K_2, s, and K_1 were varied until a good agreement between harmonic averages of results of simulations and experimental data was found. A good fit to experimental data was obtained for
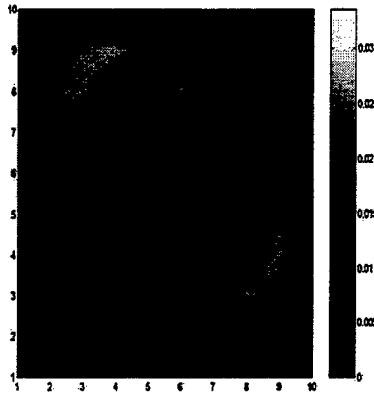
Figure 2: Time average plot for indicator functions from experimental data.

parameters $f_0 = 207$, $N = 10$, $\alpha = 0.03$, $K_2 = 2000$, $s = 5$, $K_1 = 0.0788$.

Let $p(i)$ be the pressure at time $i/2000$ obtained from experimental data or the model. In figures 2 and 3 we show the plot of time-averages

$$\chi^*_{(i,j)} = \frac{1}{20000} \sum_{i=1}^{20000} \chi_{(i,j)}(p(i))$$

of indicator functions $\chi_{(i,j)}$ on squares defined in the embedding space (an indicator function is 1 if a point is inside the square of side length l and 0 elsewhere). A grid of 10×10 indicator functions was used with $l = 2$ *psi*, their time-averages computed and assigned to nodes labelled (i,j) where i, j vary from 1 to 10. The results shown in 2, for the experimental data and 3 for the model that we found a good fit to the data are, for the sake of better visualization, linearly interpolated shaded contour-plots of the time-averages.

3.8 Control of conservative systems

In the papers [14, 15] we presented a framework for developing necessary and sufficient conditions for controllability in a class of conservative systems with drift. Systems preserving a smooth measure on the phase space, such as Hamiltonian systems of classical dynamics or incompressible flows of fluid dynamics attract a lot of interest in control theory. In [14] we describe some work on the notion of controllability in systems that are measure-preserving and possess drift. Relationship between controllability, a fundamental concept in control theory, and the concepts of integrability and ergodicity, fundamental in dynamical systems theory is addressed. The basic idea is that studying reccurence (or ergodic) properties of trajectories of the drift is key to establishing necessary and sufficient conditions for controllability in such systems. The benefit of this approach is that controllability proofs contain a constructive procedure for control. Control of Hamiltonian systems with drift is investigated for the case when the drift is integrable. Transformation of the system to action-angle coordinates is used to describe the ergodic partition of the drift. This is in turn used to obtain conditions for controllability of such systems. The key idea is that control must be capable of moving the system transverse to any set in the ergodic partition of the drift Hamiltonian vector field. Using this, additional results on controllability of more general systems are obtained.
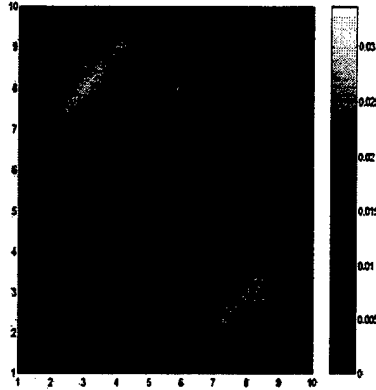
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Figure 3: Time averages for indicator functions for model with parameters that provide a good match with the experimental data.

In the paper [31] we study the controllability question for a class of discrete time nonlinear systems which arise as a discretization of a continuous time integrable Hamiltonian systems. We give necessary and sufficient condition for global controllability of these discrete time nonlinear systems under the assumption that system satisfies weak regularity condition. We also show that under these regularity condition the system is almost everywhere controllable. The result in this paper are an extension of results in [30].

3.9 Visualization of dynamical systems using harmonic analysis methods

A method for visualization of dynamical systems based on harmonic analysis was pursued in [9]. We considered a discrete-time dynamical system

$$x_{i+1} = T(x_i),$$
$$y_i = f(x_i), \tag{9}$$

where $i \in \mathbb{Z}$, $x_i \in M$, $T : M \rightarrow M$ measurable and f a smooth real function on a compact Riemannian manifold M endowed with the Borel sigma algebra. We call the function f^* the time average of a function f under T if

$$f^*(x) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} f(T^i x)$$

almost everywhere (a.e.) with respect to the measure μ on M. The time average f^* is a function of the initial state x. This function can be used to visualize invariant sets [19]. Harmonic averages of the form

$$f_\omega^* = \lim_{n \to \infty} \frac{1}{n} \sum_{j=0}^{n-1} e^{i2\pi j\omega} f(T^j(x)),$$

can be used to visualize resonances in the system (invariant sets for higher iterates of the map).
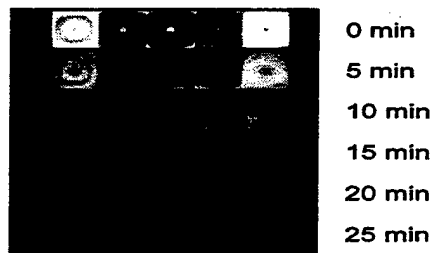
Figure 4: Time evolution of mixing in an electrohydrodynamically driven flow.

3.10 Controlled capture into resonances

In the paper [34] we propose a method to use *capture into resonance* to control the behavior of a certain class of dynamical systems. In many dynamical systems the coupling between the unperturbed system and weak periodic perturbations (wave) can be reduced to a purely resonant interaction occurring in the vicinity of a certain surface in the phase space. While resonance interaction can change invariants of the unperturbed system (e.g. energy), it is random in nature, and, consequently, is rather inefficient as a mechanism of regular transport. We propose a method to structure the resonance interaction with little additional cost. When the nominal dynamics brings the system close to a resonance surface we apply a short control pulse to force the capture of a phase point into the resonance with the wave. A captured point is transported by the wave across the energy levels. We apply the second pulse to release a phase point from the resonance when the desired energy level is achieved. As a model problem we consider dynamics of a charged *nanoparticle* in an electromagnetic field.

3.11 Uniform, resonant chaotic mixing in fluid flows

In the paper [25] which appeared in the journal *Nature*, we pursued experimental confirmation of the theory developed in [12]. Laminar flows can produce particle trajectories that are chaotic, with nearby tracers separating exponentially in time. For time-periodic, two-dimensional and steady, three-dimensional (3D) flows, enhancements in mixing due to chaotic advection are typically limited by impenetrable transport barriers that form at the boundaries between ordered and chaotic mixing regions. However, for time-dependent, 3D flows, it has been proposed theoretically [12] that completely uniform mixing is possible via a resonant mechanism called singularity-induced diffusion (SID), even if the time-dependent and 3D perturbations are infinitesimally small. It is important to establish the conditions for which uniform mixing is possible and whether or not those conditions are met in flows that typically occur in nature. In the paper [25] we present experimental and numerical studies of mixing in a laminar, weakly 3D, weakly time-periodic vortex flow. An oscillating horizontal vortex chain is generated magnetohydrodynamically; the flow is weakly 3D due to a secondary flow forced spontaneously by Ekman pumping, a mechanism common in vortical flows with rigid boundaries. As predicted, completely uniform mixing is found, only for oscillation periods close to typical circulation times. In figure 4 we present experimental evidence of fast mixing of dye in the experimental apparatus.

4 Personnel supported:

Faculty: Igor Mezić, **Postdoctoral fellow:** Dmitri Vainchtein, **Partially supported graduate students:** Umesh Vaidya, Thomas John, Zoran Levnajic, Andre Valente.

5 Interactions/transitions:

5.1 Academic interactions/Transitions

The PI and other members of the group gave a number of invited lectures on the topics of research described here, for example at Northwestern University, Caltech, SIAM Conference on Applications of Dynamical Systems, UCLA Institute for Pure and Applied Mathematics, at NOLCOS 2001, Caltech, University of Southern California, DARPA, Oberwolfach "Dynamical Systems Methods in Fluid Dynamics" meeting, US National Congress of Theoretical and Applied Mechanics etc., MIT, Boston University, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK, University of Minnessota, etc. The PI and collaborators from UTRC (Andrzej Banaszuk and Satish Narayanan) participated in two sessions at the 2000 CDC, with Narayanan and IM presenting an overview paper on flow control. The PI co-organized (with Vered Rom-Kedar) a minisymposium on mixing at the 2001 SIAM Conference on Applications of Dynamical Systems and presented a talk there on Control of Mixing. Dmitri Vainchtein and Umesh Vaidya participated in a summer program on Nonlinear Dynamics at the Technical University of Denmark. Dmitri Vainchtein presented a talk on vortex merger work at the united Technologies Research Center and also participated in Southern California Control Conferences. Umesh Vaidya gave a talk at one of these meetings as well.

5.2 Industrial interactions

There was an interaction with UTRC's Control and Dynamics group on topics in flow control. The PI interacted with Andrzej Banaszuk's group on control of combustion instabilities. Some of the model validation ideas are being extended to treat uncertainty analysis in models together with a group from United Technologies.

6 Transitions

6.1 2001

1. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Satish Narayanan. *Result:* Discussion of methods of analysis and experimentation for jet noise reduction 2. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Andrzej Banaszuk. *Result:* Shear flow control program.

6.2 2002

1. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Andrzej Banaszuk. *Result:* Shear flow control program. 2. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Mark Myers. *Result:* Planning and technical contributions to building systems modeling and control; planning and technical contributions to the uncertainty analysis program.

6.3 2003

1. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Andrzej Banaszuk. *Result:* Control of combustion instabilities presented in [11]. 2. *Performer:* I. Mezić *Customer:* United Technologies Research Center, Hartford, Connecticut. *Contact:* Dr. Mark Myers. *Result:* Planning and technical contributions to the uncertainty analysis program.

7 Honors/Awards

The PI has been awarded (jointly with Domenico D'Alessandro and Mohammed Dahleh) the George S. Axelby Outstanding Paper Award (IEEE Transactions on Automatic Control), at the CDC 2000 in Sydney for work on "Control of Mixing: A Maximum Entropy Approach".
The PI became an Editor for Physica D: Nonlinear Phenomena.
The PI became an Editor for Journal of Applied Mechanics.

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